AP Calculus BC

Areas in a Plane

1) $y = x^2$ $y = 4x$

$x^2 = 4x$

$x^2 - 4x = 0$

$x(x-4) = 0$

$x = 0$ $x = 4$

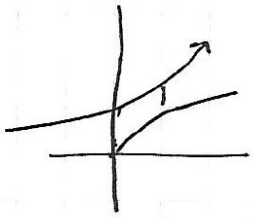
$\int_0^4 [x^2 - 4x]\,dx$

$\left[\frac{1}{3}x^3 - 2x^2\right]\Big|_0^4$

$\frac{64}{3} - 32$

Area $= 32 - \frac{64}{3}$

2) $y = e^x$ $y = \sqrt{x}$ $y = 0$ $x = 1$



$\int_0^1 [e^x - x^{1/2}]\,dx$

$\left[e^x - \frac{2}{3}x^{3/2}\right]\Big|_0^1$

$\left(e - \frac{2}{3}\right) - 1 = \boxed{e - \frac{5}{3}}$

3) $y = 4 - x^2$ $y = -4$

$4 - x^2 = -4$

$-x^2 = -8$

$x = \pm 2\sqrt{2}$

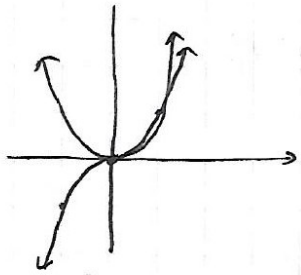
$\int_0^{2\sqrt{2}} [4 - x^2 - (-4)]\,dx$

$\int_0^{2\sqrt{2}} [8 - x^2]\,dx$

$\left[8x - \frac{1}{3}x^3\right]\Big|_0^{2\sqrt{2}}$

$\boxed{16\sqrt{2} - \frac{1}{3}(2\sqrt{2})^3}$

4) $y = x^3$ $y = x^2$



$\int_0^1 [x^2 - x^3]\,dx$

$\left[\frac{1}{3}x^3 - \frac{1}{4}x^4\right]\Big|_0^1$

$\boxed{\frac{1}{3} - \frac{1}{4}}$

5) $x = y^2$ $x - y = 2$

$x = y + 2$

$y^2 = y + 2$

$y^2 - y - 2 = 0$

$(y-2)(y+1) = 0$

$y = 3$ $y = -1$

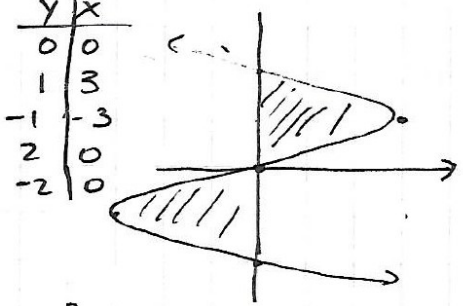
$\int_{-1}^{3} [y + 2 - y^2]\,dy$

$\left[\frac{1}{2}y^2 + 2y - \frac{1}{3}y^3\right]\Big|_{-1}^{3}$

$\boxed{\left[\frac{9}{2} + 6 - 9\right] - \left[\frac{1}{2} - 2 + \frac{1}{3}\right]}$

6) $x = 4y - y^3$ $x = 0$

y	x
0	0
1	3
-1	-3
2	0
-2	0



$2\int_0^2 [4y - y^3]\,dy$

$2\left[2y^2 - \frac{1}{4}y^4\right]\Big|_0^2$

$2[8 - 4] = \boxed{8}$

7) $x+y=3 \quad y+x^2=3$

$y=-x+3 \quad y=-x^2+3$

$-x+3=-x^2+3$

@ $x=0, 1$

$\int_0^1 [-x^2+3-(-x+3)]\,dx$

$= 0.167$

8) $y=\sqrt{x} \quad y=-x \quad x=1 \quad x=4$

$\int_1^4 (\sqrt{x}+x)\,dx = 12.167$

9) $y_1=\cos x \quad y_2=x^2+10x+16$

$Y_1=Y_2$ @ $x=-7.979 \rightarrow A$

$x=-2.082 \rightarrow B$

$\int_A^B [Y_1-Y_2]\,dx = 36.099$

10) $y_1=\sin(4x) \quad y_2=1+\cos\left(\frac{x}{3}\right)$

$\int_0^{\pi} [Y_2-Y_1]\,dx = 5.7397$

11) $y_1=x^2-4x+2 \quad y_2=2$

$Y_1=Y_2$ @ $x=0, x=\pm 2$

$\int_{-2}^{0} (Y_1-Y_2)\,dx + \int_0^2 (Y_2-Y_1)\,dx$

$= 8$

12) $f(x)=|x^2-6x+5| \quad x=0, x=7$

$\text{Area} = \int_0^1 (x^2-6x+5)\,dx - \int_1^5 (x^2-6x+5)\,dx + \int_5^7 (x^2-6x+5)\,dx$